ARRAY ANTENNAS COUPLING MODEL FOR MULTIMODE RADIATORS

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Abstract: *This paper introduces a mathematical model of coupling and radiation field in array antennas that allows an easy antenna analysis and design. The model is based in the linear behaviour of antennas and in the option to separate the radiated field structure in a reduced number of modes. Special attention can be paid to resonant printed antennas. Printed antenna radiation field can be described through a reduced number of modes, usually one or two, to reach high accuracy in the radiated field computation.*

1. Introduction

Array antenna design has not usually taken into account coupling between elements. The field radiated by each element of an N element array is assumed equal to the free space radiation. Actual input impedance and radiated field from each element in the array can differ very much from the single element model. Input impedance can be computed through an N-port network model described by impedance (**Z**) or scattering (**S**) matrix. Radiated field can be computed if we know the active radiated field from each element that takes into account the complete array influence in each element radiated field, as presented by Mailloux [1]. In receiving antennas the model would be the same, giving an equivalent active diagram and active output impedance that depends on the loading circuit. In large arrays, most elements present similar conditions: their class, position and load. That situation provides equal active impedance and diagram for most elements in the array except the extreme ones. In [2], Wasylkiwskij shows the relation between input impedance and radiated field for Minimum Scattering Antennas (MSA). Many times, real antennas have been assumed as MSA with good results [3]. In other cases, that assumption is not as fortunate and relation between input coupling matrix (**Z**) and active radiated field is not so clear.

One of the most important questions in the design of antenna diagram or in the active and adaptive antenna behaviour is if the coupling can be modelled by an *NxN* coupling matrix (**C**) as described in most studies of adaptive arrays [4]. In this paper, a matrix model is presented for array antennas fed through linear networks. The model is based in the several active modes of the array elements and active diagram computation as a function of these modes. If "*k*" independent modes can describe the electromagnetic behaviour of each element, *Nxk* modes will be needed to define all the array radiation characteristics. An *Nx(k+1)* square matrix will describe the array behaviour. This description is independent of the feeding network, feeding distribution or transmission-reception application [5].
For many printed patches only one resonant radiation mode can be used to describe the radiated field of each element. When this happens a *2Nx2N* matrix describes the complete array model. Based in this model, and applying the feeding network parameters, a coupling **C** matrix can be obtained to compensate the design model from the influence of element coupling. As an example, a printed antenna of rectangular patches as that described by Pozar in [6], allows us to demonstrate how accurate the matrix model introduced in this paper is.

2. Radiated/received field model for an individual antenna

2.1. Transmission model

The antenna electrical behaviour can be defined by its input impedance and its radiated field (Eq. 1)

$$\overline{E}_{rad} ? v_e \hat{e}(?,?)F(?,?)\frac{\exp(? jkr)}{r} \tag{1}$$

where v_o and i_o are the feeding voltages and currents, v_e is a voltage proportional to the input current, $F?,??$ is the radiation pattern, $\hat{e}?,??$ is the polarisation vector. The antenna can also be seen as a function of its scattering matrix. Then the radiated field and the radiated power can be expressed as Eq. 2 and Eq. 3

$$e_{rad}(?,?) ? b_e \sqrt{2?_o}\; F_s(?,?)\frac{\exp(? jk_0 r)}{r}\hat{e}_e(?,?) \tag{2}$$

$$P_{rad} ? ?|a|^2 ?1 ? |S_a|^2 ? ? |b_e|^2 \int_{4?} |F_s ?,??|^2 d? \tag{3}$$

S_a represents the reflection coefficient at the input defined respect Z_o, b_e is a power wave proportional to the amplitude and phase of the input power wave (a) and F_s is the radiation pattern respect the S parameters; the input reference impedance is Z_o while the output reference impedance is $?_o$=120?. The antenna gain can be expressed by using the normalised radiation pattern and the antenna efficiency as Eq. 4

$$G?,??? ? 4?|F?,??|^2 ? \frac{|S_e|^2}{1 ? |S_a|^2}4?|F?,??|^2 \tag{4}$$

2.2. Reception model

When the antenna works in receiving way the equivalent surface A(?,?) represents the amount of power taken by the antenna. If S_a and Z_a represent the impedance and reflection coefficient of the circuit, then a power wave (b) can be extracted at the input port.

$$P_{dis} ? |a_e|^2 \frac{|S_r|^2}{1 ? |S_a|^2} ? |\hat{e}_i ?\hat{e}_e ?,??|^2 \frac{1}{2?_0}|E_i|^2 \frac{|S_r|^2}{1 ? |S_a|^2} ?^2 |F?,??|^2 ; A?,??? ? ?^2 |F?,??|^2 \tag{5}$$

A reception power wave proportional to the impinging field can be defined as the square root of (5)

$$a_e ? ?\hat{e}_i ?\hat{e}_e ?,???\frac{1}{\sqrt{2?_0}}|E_i|?|F?,??|; P_{dis} ? |a_e|^2 \frac{|S_r|^2}{1 ? |S_a|^2} \tag{6}$$

If the antenna equivalent surface is written as a function of the antenna gain and the reciprocity principle is applied then S_e=S_r

$$A?,??? \frac{?^2}{4?}G?,??; \quad Z_e ? Z_r \quad ; \qquad S_e ? S_r \tag{7}$$

3. Radiated/received field model for an array antenna

A first approach of a model to take into account the previous effects has been proposed in [5, 7]. This new network takes into account the decomposition of the current distribution in multiple characteristic modes. Then any N-array antenna can be represented as a ((k+1)?N)-network. The new network has one input and k output ports corresponding to any of the radiating modes of each antenna (by input ports we mean any of the actual probes of the array while by output ports we mean fictitious ports representing any radiating function). Figure 1 shows the new $1+n$ port network (1 corresponding to the antenna input and n to the radiation modes). When the radiating elements are resonant microstrip antennas, only one radiating mode may be considered resulting in a 2N-port network. This network is represented in Figure 2. The terminals at the left side of the 2N-port network represent physical probes of the antenna that can be directly measured while the ones at the right side allow us to define the radiation functions. They will never be charged since they represent ideal radiating (b_e) or receiving (a_e) antennas. Then the matrix equation relating previous variables is given in (8)

$$\begin{bmatrix}\mathbf{b}\\ \mathbf{b}_e\end{bmatrix}_{(2N\times1)} = \mathbf{S}_{(2N\times2N)}\begin{bmatrix}\mathbf{a}\\ \mathbf{a}_e\end{bmatrix}_{(2N\times1)} \Rightarrow \begin{cases}\mathbf{b} = \mathbf{S}_a\cdot\mathbf{a} + \mathbf{S}_r\cdot\mathbf{a}_e\\ \mathbf{b}_e = \mathbf{S}_e\cdot\mathbf{a} + \mathbf{S}_s\cdot\mathbf{a}_e\end{cases} \qquad (8)$$

where S_a represents the reflection coefficient, vector $\mathbf{S_e}$ represents the transmission parameter for each mode, $\mathbf{S_r}$ represents reception coupling for any of the defined modes and matrix $\mathbf{S_s}$ indicates the scattered field by each mode.

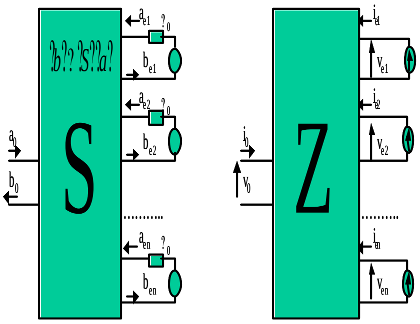

Figure 1: Equivalent *n+1* port network for an individual antenna with several radiation modes

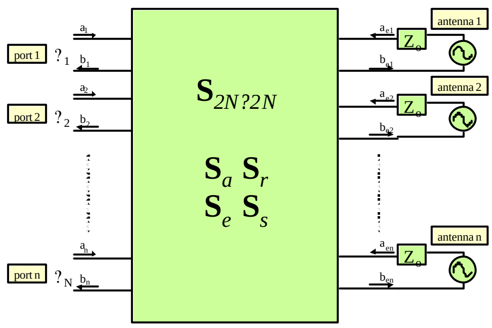

Figure 2: Equivalent 2N port network for an array antenna with only one radiation mode

When operating in a transmitting way, the array is fed through a set of generators with an equivalent incident wave $\mathbf{a}_{g(N\times1)}$ and source reflection coefficient $\Gamma_{g(N\times N)}$. The total radiated field can be expressed as the sum of the contributions from any of the radiating elements. This can be written in vector form as the dot product of the steering vector and the transmitting power wave

$$\mathbf{e}_{rad}(\theta,\phi) = \left[\mathbf{d}^T(\theta,\phi)\cdot\mathbf{b}_e\right]\sqrt{2Z_o}\,\frac{\exp(-jk_o r)}{r} = \mathbf{d}^T(\theta,\phi)\cdot\mathbf{C}_e\cdot\mathbf{a}_g\sqrt{2Z_o}\,\frac{\exp(-jk_0 r)}{r};\ \mathbf{C}_e = \mathbf{S}_e\cdot(\mathbf{I}-\Gamma_g\cdot\mathbf{S}_a)^{-1} \qquad (9)$$

d is the steering vector in (θ,ϕ) direction. Each term of matrix $\mathbf{C}_e$ (C_{ij}) represents the amount of signal coupled from antenna *j* to *i*. The radiation intensity can be expressed as the following dot product

$$U(\theta,\phi) = \frac{r^2}{2\eta_0}\left|\bar{E}_{rad}\right|^2 = \mathbf{a_g^H C_e^H d^*}\cdot\mathbf{d^T C_e a_g} = \mathbf{a_g^H}\cdot\mathbf{M}_e\cdot\mathbf{a_g} \qquad (10)$$

Emission matrix depends on the radiation directions but not on the way of feeding the array. The analysis of this matrix allows us to determine the maximum radiation directions and the blind directions. This can be done by maximising the following generalised Rayleigh quotient

$$\frac{U(\theta,\phi)}{P_{dis}} = \frac{\mathbf{a_g^H C_e^H d^*}\cdot\mathbf{d^T C_e a_g}}{\mathbf{a_g^H}(\mathbf{I}-\mathbf{S_c S_c^*})\mathbf{a_g}} = \frac{\mathbf{a_g^H M_e a_g}}{\mathbf{a_g^H}(\mathbf{I}-\mathbf{S_c S_c^*})\mathbf{a_g}} \qquad (11)$$

4. Application: equivalent scattering and coupling model for an array antenna

Two examples are shown to validate previous model. First a simulation model for a linear array with 12 elements fed through a slot as shown in figure 3. Figures 4 and 5 show the corresponding coupling parameters. A square array with four elements has been built to compare simulation results with measurements. These are shown in Table I [5]

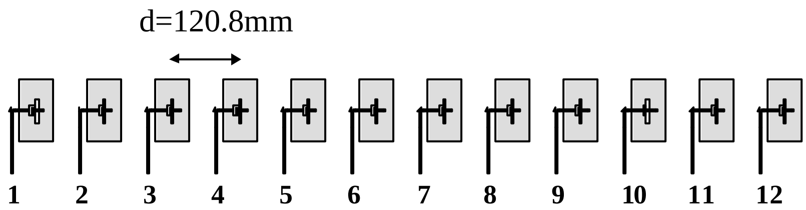

Figure 3: Array with 12 elements in the Eplane at 1800 MHz band

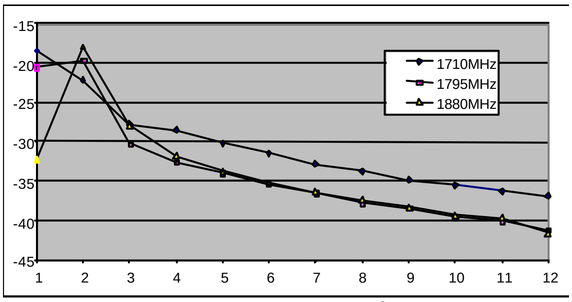

Figure 4: s_{1i} parameters between the first element and the other ones

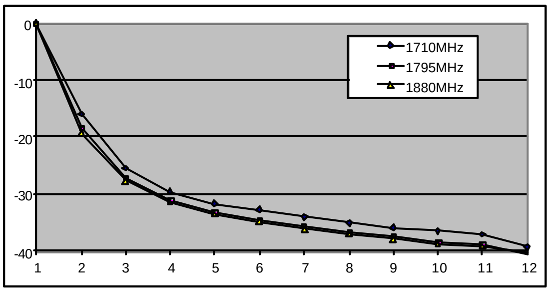

Figure 5: Radiation coupling parameters (S_e) between the first element and the other 11 elements in the array

TABLE I

Measured (MEAS), Calculated (CM) and Full Simulation (SIM) S-parameters

S-parameter	MEAS	CM	SIM
S12	-20.8dB$_{147^{o}}$	-23.8dB$_{153^{o}}$	-23.8dB$_{152^{o}}$
S13	-24.9dB$_{117^{o}}$	-28.6dB$_{142^{o}}$	-28.7dB$_{140^{o}}$
S14	-29.5dB$_{55^{o}}$	-33.4dB$_{52^{o}}$	-33.3dB$_{51^{o}}$
S23	-29.3dB$_{32^{o}}$	-33.4dB$_{52^{o}}$	-33.3dB$_{51^{o}}$
S24	-24.5dB$_{138^{o}}$	-28.6dB$_{141^{o}}$	-28.7dB$_{140^{o}}$
S34	-21.9dB$_{142^{o}}$	-23.8dB$_{153^{o}}$	-23.9dB$_{156^{o}}$

2×2 patch array at 3.5 GHz [5]

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